Direct Analysis Method – Part I

Solver Prerequisites for Accuracy of Solution

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The Direct Analysis Method

*Specifications for Structural Steel Buildings***,** ANSI/AISC 360-10, published by the American Institute of Steel Construction, outlines the Direct Analysis Method (DM) and specifies its use in the design of steel structures. DM is defined by AISC as "a design method that captures the effects of residual stresses and initial out-of-plumbness of frames by reducing stiffness and applying notional loads in a second order analysis." DM also accounts for uncertainty in material strength and stiffness as well as stiffness reduction due to inelasticity. Flexural, shear and axial deformations are considered, and, when necessary, included in DM.

According to AISC, "recent research has shown that the AISC direct analysis method … is the best approach to cover all relevant response effects."

Second Order Analysis

Second order analysis, as required by the direct analysis method, is a means used by engineers to uncover deformations and forces that would otherwise be overlooked in conventional linear static analysis. Two common second order effects include P-Δ effects and the less-known P-δ effects.

P-Δ effects, also known as 'big' P-delta, refer to the increase in moment experienced by a fixed connection at the i^{th} end of a member when the j^{th} end is loaded axially while simultaneously undergoing lateral translation. Put more simply, one can imagine a cantilever beam with both axial and transverse loading. Simple static analysis tells us that the fixed end only experiences a moment equal to the magnitude of the transverse loading multiplied by the length of the member. However, P-Δ analysis reveals that an additional moment is created due to the application of the axial load (P) no longer lining up with the support (it is displaced by an amount, Δ). This can be seen in Figure A.

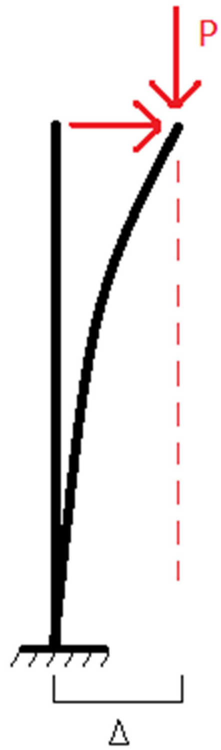

Figure A: P-Δ Effects

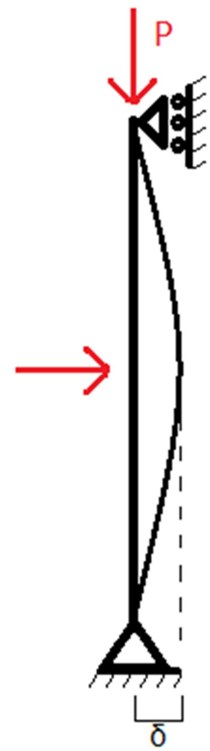

Figure B: P-δ Effects

Figure B represents a case where P-δ effects could be considered. Rather than increase the end forces, P-δ effects (or 'little' P-delta) cause an increase in the member internal forces. This is a result of an axial force acting through lateral translation of a member relative to its own longitudinal axis.

Both P-Δ and P-δ may or may not be negligible in the analysis of a structure. It is the job of a structural engineer, in consultation with ANSI/AISC 360-10, to decide if they should be included.

S-FRAME

New in S-FRAME version 10.0 is the ability to use the direct analysis method, as outlined in ANSI/AISC 360-10. Unlike some software, S-FRAME requires only one solution run to perform the direct analysis method. Additionally, S-FRAME reports the calculated value of B_2: the ratio of second order effects to first order effects. B_2 is useful for engineers trying to determine whether consideration of first order effects alone might be adequate for their analysis.

As mentioned, the use of DM requires a rigorous second order analysis in order to meet the code's requirements. ANSI/AISC 360-10 provides two benchmark cases that allow us to "determine whether an analysis procedure meets the requirements of rigorous second-order analysis adequate for use in the direct analysis method." These cases are quite similar to the P-Δ and P-δ examples presented above. This should come as no surprise due to the role these second-order effects play in the direct analysis method. If we model and analyze these cases in S-FRAME, it should be possible to demonstrate that S-FRAME's implementation of P-Δ and P-δ analysis is rigorous and adequate. It is worth noting that these are designed to be first level checks only and that additional checks are available[1].

ANSI/AISC 360-10 requires analysis and theoretical values to be within 3% agreement for moments and 5% for deflections.

Case One

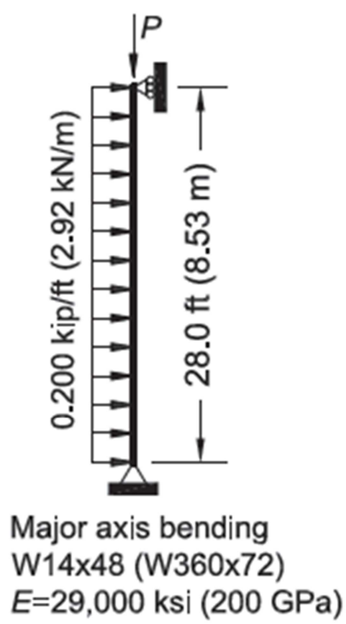

Figure C: Case One

Case One, as seen in Figure C, is designed to reveal errors in calculating P-δ effects in a given analysis method. As there is no lateral translation of either end, P-Δ effects do not enter consideration. In this case, the uniformly distributed load causes the member to deform to the right. This deformation results in the additional internal member forces discussed earlier. As this case investigates moment and deflection at midspan, we modeled it using two beam elements of equal length. This way, a node was located at our

[1] Kehler et al. (2010), Chen and Lui (1987), and McGuire et al. (2000)

point of interest. Table 1 compares the theoretical deflections and moments to those calculated by S-FRAME for various values of the axial load, P.

Table 1: Results Comparison for Case One

Axial Load (kips)		Theory	S-FRAME	Difference (%)
0	Moment at Midspan (kp-in)	235	235.200	0.09
	Defl. at Midspan (in)	0.202	0.201	0.26
150	Moment at Midspan (kp-in)	270	269.707	0.11
	Defl. at Midspan (in)	0.23	0.230	0.08
300	Moment at Midspan (kp-in)	316	315.610	0.12
	Defl. at Midspan (in)	0.269	0.268	0.37
450	Moment at Midspan (kp-in)	380	379.647	0.09
	Defl. at Midspan (in)	0.322	0.321	0.31

Figures D and E display graphically the results of Case One, where the blue bars represent the difference between S-FRAME's computation and the theoretical value. The horizontal red lines indicate the maximum acceptable difference as stated in ANSI/AISC 360-10. Clearly, the moments and deflections calculated by S-FRAME are well within the limits of agreement set out by the code. In fact, in this case, S-FRAME differed from theory by no more that 0.37%, or less than 1 in 250.

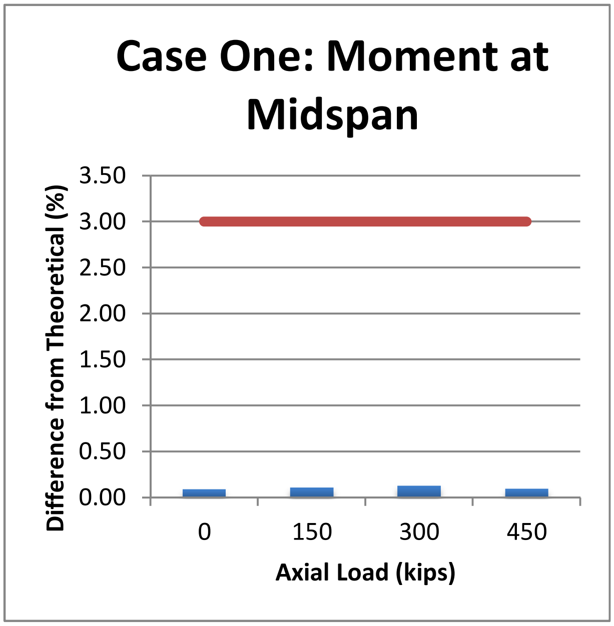

Figure D: Difference in Moment (in percent) between Theory and S-FRAME

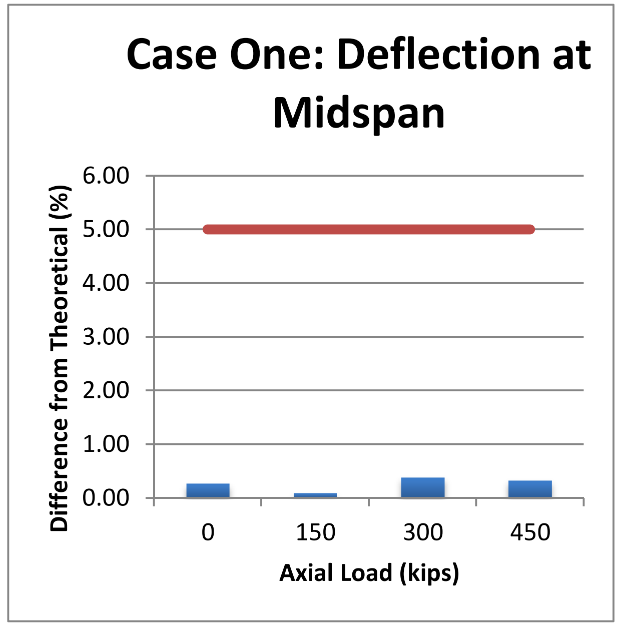

Figure E: Difference in Deflection (in percent) between Theory and S-FRAME

Case Two

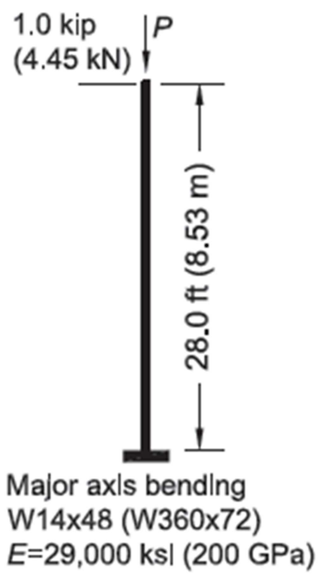

Figure F: Case Two

Case Two, as seen in Figure F, is designed to reveal errors in calculating P-Δ *and* P-δ effects in a given analysis method. In this case, the combination of loads applied to the free end of the cantilever causes additional end forces as well as additional internal member forces. For Case Two, nodes were already located at the points of interest, the base and tip, and, as such, it was not necessary to use more than one beam element. Table 2 compares the theoretical deflections and moments to those calculated by S-FRAME for various values of the axial load, P.

Table 2: Results Comparison for Case Two

Axial Load (kips)		Theory	S-FRAME	Difference (%)
0	Moment at Base (kp-in)	336.00	336.000	0.00
	Defl. at Tip (in)	0.91	0.907	0.01
100	Moment at Base (kp-in)	470.00	470.141	0.03
	Defl. at Tip (in)	1.34	1.341	0.07
150	Moment at Base (kp-in)	601.00	601.809	0.13
	Defl. at Tip (in)	1.77	1.765	0.26
200	Moment at Base (kp-in)	856.00	852.705	0.38
	Defl. at Tip (in)	2.60	2.584	0.63

Figures G and H display graphically the results of Case Two, where the blue bars represent the difference between S-FRAME's computation and the theoretical value. The horizontal red lines indicate the maximum acceptable difference as stated in ANSI/AISC 360-10. Again the moments and deflections calculated by S-FRAME are well within the limits of agreement set out by the code. In Case Two, S-FRAME differed from theory by no more than 0.63% or less than 1 in 150.

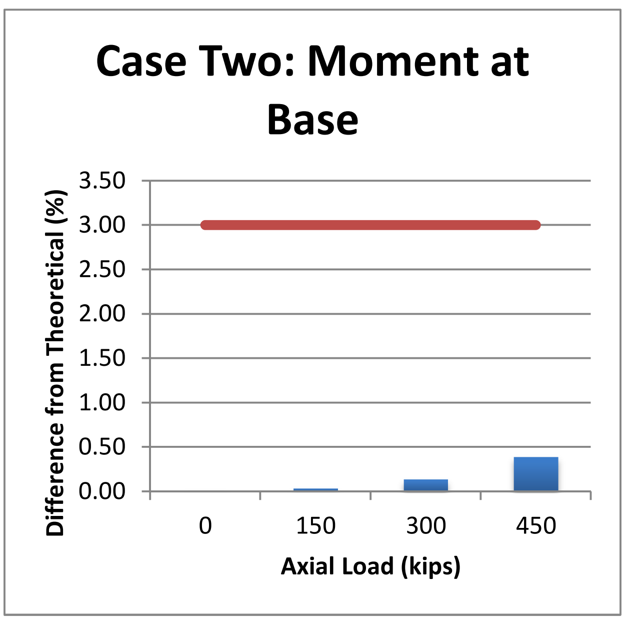

Figure G: Difference in Base Moment (in percent) between Theory and S-FRAME

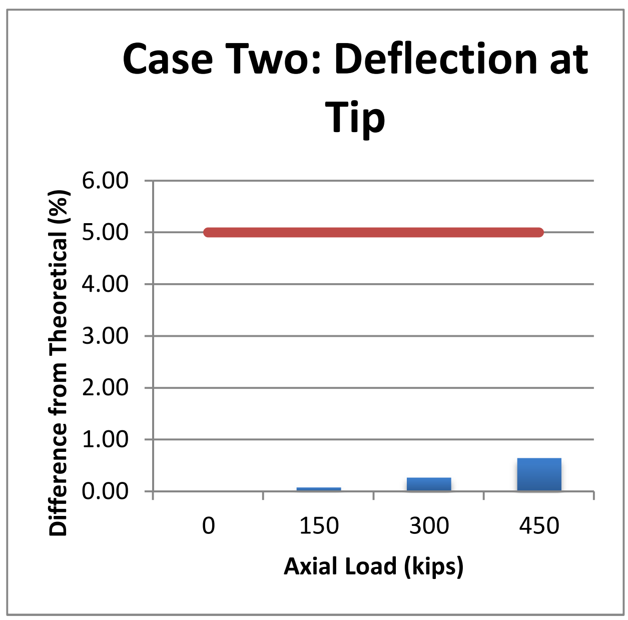

Figure H: Difference in Tip Deflection (in percent) between Theory and S-FRAME

Summary

Consistently, S-FRAME's implementation of second-order effects P-Δ and P-δ, provides results which differ from theory by well less than one percent. This is well within the 3-5% cut off for acceptable analysis methods outlined in ANSI/AISC 360-10. Taking into account typical uncertainties associated with material properties and structure demands, it is safe to conclude that the difference between theoretical values and S-FRAME's computations is negligible. Furthermore, this indicates that S-FRAME is a reliable tool for calculating second-order effects. S-FRAME's P-Δ and P-δ analysis is accurate and rigorous and more than sufficient for use alone or as a part of the direct analysis method.

Appendix 1 – Critical Buckling Loads

S-FRAME's P-Delta buckling analysis can calculate the load that will cause a structure or member to buckle. In this appendix, the results of S-FRAME's buckling analysis are compared to those predicted by theory. The same two cases as earlier will be used to compare these results.

Case One

The theoretical buckling load for the simply supported beam can be calculated as follows:

Elastic Modulus	E = 29000 ksi
Moment of Inertia	I = 484 in^4
Length	L = 28 ft
Effective Length Factor	$L_e = 1$

$P = E \times I \times \pi^2 / (L \times L_e)^2$ = **1227.056** kips

S-FRAME calculates the buckling load to be **1236.735** kips, a 0.789% difference. Note that, as before, two analysis members were used for this beam.

Case Two

The theoretical buckling load for the cantilever beam can be calculated as follows:

Elastic Modulus	E = 29000 ksi
Moment of Inertia	I = 484 in^4
Length	L = 28 ft
Effective Length Factor	$L_e = 2$

$P = E \times I \times \pi^2 / (L \times L_e)^2$ = **306.764** kips

S-FRAME calculates the buckling load to be **309.184** kips, a 0.789% difference. Note that, as before, one analysis member was used for this beam.

Appendix 2 - Buckling of a Frame

It has been shown that S-FRAME's P-Δ buckling analysis is accurate for calculating the buckling loads of cantilever and simply supported columns for which there are closed-form analytical solutions available. However, in practice structural systems are usually more complex than single columns. This appendix examines the critical buckling loads of a simple 2D frame example by Chen and Lui's *Structural Stability: Theory and Implementation* referenced by *ANSI/AISC 360-10: Specifications for Structural Steel Buildings*.

The frame will be considered for two support cases: with and without sway allowed. For the sway prevented case, supports were added at the top of each column which prevented any in-plane lateral displacement only. Two load cases will be considered for each support case, as shown in Figures A and B, for a total of four cases. These four cases are outlined in Table 1

Each frame is 10 feet in height and 15 feet in span.

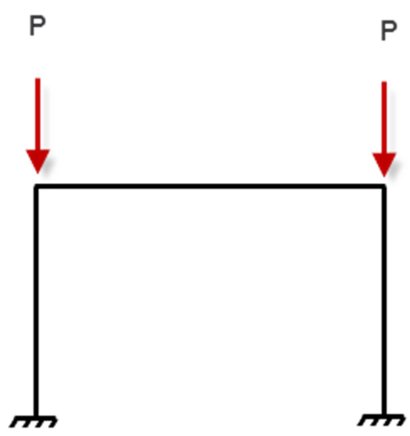

Figure A: Type 1 Loading

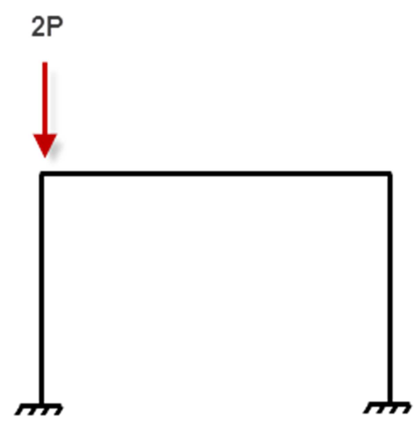

Figure B: Type 2 Loading

Table 3: Support Conditions and Loading

	Sway	Loading
Case 1	Permitted	Type 1
Case 2	Permitted	Type 2
Case 3	Not Permitted	Type 1
Case 4	Not Permitted	Type 2

For the entirety of the frame, a W14x48 section was used with the section and material properties as below. As mentioned, the length of the columns is 10 feet.

Moment of Inertia; $I_x = 484\ in^4$;

Elastic Modulus; $E = 29000$ ksi;

Length; $L = 10$ ft;

Applying Euler's formula, we can calculate the critical buckling load, P_e, for a simply supported W14x48 column, 10 feet in length.

$$P_e = \pi^2 \times E \times I_x / L^2 = \mathbf{9620.123} \text{ kips;}$$

Chen and Lui provide critical buckling loads for the four cases discussed earlier as a fraction of the critical buckling load for a simply supported column. The critical buckling loads for each case can then be calculated as follows:

$$P_{cr1} = P_e \times 0.6694 = \mathbf{6439.710} \text{ kips}$$

$$P_{cr2} = P_e \times 0.661 = \mathbf{6358.901} \text{ kips}$$

$$P_{cr3} = P_e \times 2.407 = \mathbf{23155.635} \text{ kips}$$

$$P_{cr4} = P_e \times 1.314 = \mathbf{12640.841} \text{ kips}$$

Using four analysis members per physical member, the four cases were modeled in S-FRAME and run through P-Δ buckling analysis. The results showed very good agreement to those predicted by theory. This can be seen in Table 2.

Table 4: Critical Buckling Load in Kips

	Theory	**S-FRAME**	**Difference in Percent**
Case 1	6439.7	6408.5	0.49
Case 2	6358.9	6251.7	1.71
Case 3	23155.6	23217.7	0.27
Case 4	12640.8	12709.7	0.54

It can be concluded that S-FRAME's P-Δ buckling analysis is very accurate in determining the critical buckling loads of frames as well as columns. In the cases examined, S-FRAME's results differed from theory by no more than two percent.